Power output calculator of a HAWT with tubercles using BEM Theory

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ABSTRACT

A horizontal axis wind turbine (HAWT) with tubercles was analyzed using the Blade Element Momentum (BEM) Theory. Formulas based from the Original BEM Theory, Wilson-Walker Method, Glauert's Empirical Formula, and Buhl's Theory were used in the prediction of the power output. The results were then evaluated and compared to the actual wind tunnel test results presented in the literature. The Buhl's theory was considered to be the most accurate method in predicting the power output since it had the lowest mean absolute percentage difference (MAPD) and mean absolute deviation (MAD), with values of 17.21% and 1.63 Watts, respectively.

Keywords: Biomimetics, Renewable energy, wind turbine, BEM theory

METHODOLOGY

This paper adapted the blade design and experimental results from the study of Alsultan [8], as seen on Figure 2. The design was based on a straight blade at first. Throughout the blade span, the chord lengths were altered to transform the smooth edge into sinusoidal protuberances that resembled humpback whales' tubercles. The NACA4412 airfoil was used in each cross section of the blade.

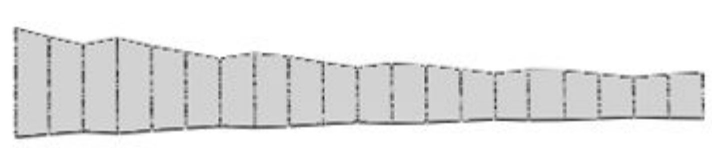

Figure 2: Tubercle Blade

The starting parameters were loaded into the code in Microsoft Excel to determine the power output. Various writers have also employed the same methods in the aerodynamic analysis of various HAWT. The results were then compared to the results from the wind tunnel experiment done on the same blade design. The BEM theory integrates the one-dimensional momentum theory and the two-dimensional blade element theory into one theory. The power output can be derived from the entire torque which can be determined by using the trapezoidal rule of integration to the power output.

$$P = T\Omega$$

where T was the total torque for the entire blade length and Ω was the angular speed of the rotor.

Original BEM Theory

The axial induction factor (a) and tangential induction factor (a') were two important values needed in the calculation of the power output. It can be derived by combining concepts from the blade element theory and momentum theory.

$$a = \frac{1}{\left[\frac{4\sin^2\emptyset}{\frac{Bc}{2\pi r}(C_L cos\emptyset + C_D sin\emptyset)} + 1\right]}, \quad a' = \frac{1}{\left[\frac{4\sin\emptyset\cos\emptyset}{\frac{Bc}{2\pi r}(C_L sin\emptyset - C_D cos\emptyset)} - 1\right]}$$

The original BEM Theory had no correction factors integrated in its formula. Many corrections had been proposed to increase the accuracy of the calculations. One of the factors that could be incorporated on the formulation was the Prandtl's tip loss factor (F).

$$F = \frac{2}{\pi}\cos^{-1}\left\{e^{\left[\frac{-B(R-r)}{2r\sin\emptyset}\right]}\right\}$$

With F defined, the axial and tangential induction factors became

$$a = \frac{1}{\left[\frac{4F\sin^2\emptyset}{\frac{Bc}{2\pi r}(C_L cos\emptyset + C_D sin\emptyset)} + 1\right]}, \quad a' = \frac{1}{\left[\frac{4F\sin\emptyset\cos\emptyset}{\frac{Bc}{2\pi r}(C_L sin\emptyset - C_D cos\emptyset)} - 1\right]}$$

Wilson-Walker Method

In 1984, Wilson reported the Wilson-Walker Method, which incorporated the Spera's correction for the thrust coefficient based on the axial induction factor. If a became greater than the critical axial induction factor value, ac, which was 0.2, the following equations could be used,

$$a = \frac{1}{2}\left\{2 + K[1-2a_c] - \sqrt{[2+K(1-2a_c)]^2 + 4[Ka_c{}^2 - 1]}\right\}, \quad K = \frac{4F\sin^2\emptyset}{\sigma_r C_n}$$

Glauert's Empirical Formula

Glauert reported in 1926 that experimental results showing the thrust coefficients to be invalid if the axial induction factor exceeds approximately 0.4. Hence, Glauert came up with an empirical formula which addresses the validity of the numerical solution.

$$a = \frac{0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_T)}}{F}, \quad C_T = \frac{F\sigma_r(1-a)^2 C_n}{\sin^2\emptyset}$$

Buhl's Theory

Buhl[7] derived a new equation for the thrust coefficient that solved the numerical instability commonly experienced in solving the BEM Theory iteratively.

$$a = \frac{18\sigma C_n + 36F^2\sin^2\emptyset - 40F sin^2\emptyset - 6\sqrt{18F\sigma_r C_n\sin^2\emptyset + 36F^4\sin^4\emptyset - 48F^3\sin^4\emptyset}}{2(9\sigma_r C_n - 50F sin^2\emptyset + 36F^2\sin^2\emptyset)}$$

INTRODUCTION

Wind turbine technology had been advancing, and a few researchers had already made significant advancements in wind turbine design. The correlation of wind turbine blades to the flippers of a humpback whale was one of the most notable of all the developments in wind turbine technology. Among the big baleen whales, the humpback whale (*Megaptera novaeangliae*) was unrivaled in its ability to perform aquatic acrobatics to acquire prey despite its enormous size. Humpback whales have wing-like flippers with tubercles, which are rounded scalloped bumps (see Figure 1). The tubercles' aerodynamic potential has been demonstrated in previous investigations. Tubercles could potentially boost power generation, eliminate lift loss, and reduce noise. The Blade Element Momentum (BEM) Theory is one of the oldest yet most effective ways in assessing HAWT loads. For the past few years, researchers and wind turbine designers have optimized and modified the BEM theory into various windmill brake state models such as the Wilson-Walker method, Glauert's empirical formula, and Buhl's theory, all of which incorporate various corrections to improve prediction precision.

Figure 1. Tubercles on the flipper of a humpback whale (*Megaptera novaeangliae*) and their novel application on wind turbines.

The BEM theory had been studied by wind turbine researchers since its inception, and marine biologists had been attracted by the aerodynamic promise of humpback whales' tubercles, but the combination of those two fields develop a new concept that had been examined only on a handful of studies.

RESULTS AND DISCUSSIONS

The results obtained from the four calculation methods were validated by comparing them to the experimental results from the wind tunnel experiment performed on the same biomimetic blade. The biomimetic blade from the small HAWT were adapted from the study conducted by Alsultan.

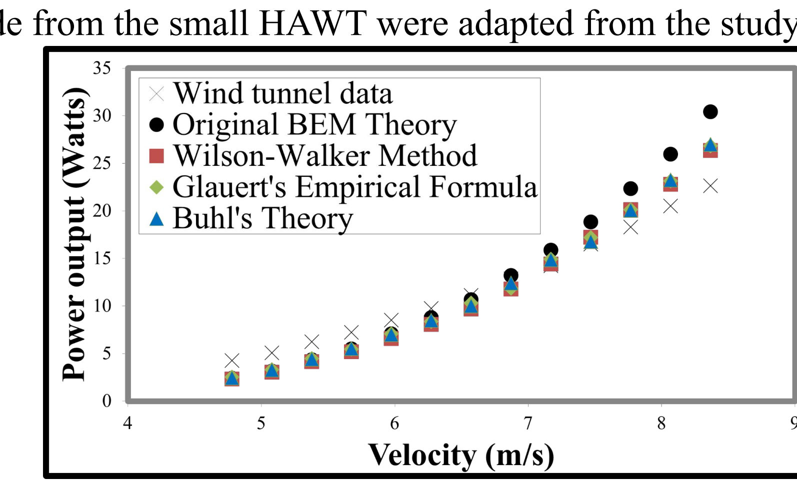

Figure 3. Power output predicted by BEM Theory compared with actual wind tunnel data

Figure 3 compared the numerical results from the four sets of formula to the experimental data taken from the wind tunnel test from the literature. As can be seen from the graph above, the numerical results from the original BEM Theory were the most dissimilar as compared to the other three brake state models. Results from the original theory started to drift away with the increase in velocity, while the other three models stayed close with each other. This particular difference was expected since the original BEM Theory did not include factors and corrections, especially the Prandtl's tip loss factor, in the calculation of the axial and tangential induction factor. To properly quantify the difference between the numerical and experimental results, the mean absolute percentage percentage differences (MAPD) and mean absolute deviations (MAD) were solved. The Buhl's theory was considered to be the most accurate method in predicting the power output since it had the lowest MAPD and MAD, with values of 17.21% and 1.63 Watts respectively.

CONCLUSIONS

This paper investigated a HAWT with tubercles by identifying the power output using the calculation frameworks based from the BEM Theory. Formuls based from the original BEM Theory and three brake state models (Wilson-Walker method, Glauert's empirical formula, and Buhl's Theory) were used in the computations. The Buhl's theory was considered to be the most accurate method in predicting the power output since it had the lowest MAPD and MAD, with values of 17.21% and 1.63 Watts respectively. These findings proved that power output prediction on HAWT with tubercles were more accurate when formulas used were integrated with corrections like the Prandtl's tip loss factor.